

# Coupling Efficiency of Butt-Joined Isotropic and Anisotropic Single-Mode Slab Waveguides

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**Abstract**—Studied is the effect of axial displacement and angular misalignment on power-coupling efficiency of a butt-joint between an isotropic and an anisotropic single-mode slab waveguide. The power-coupling coefficient is formulated by means of the boundary conditions at the interface of the butt-joint and the orthogonality relations between the modes in the outgoing waveguide. It is found from the numerical results that proper amounts of angular misalignment and axial displacement remarkably suppress transmission losses when the material coordinate system of the anisotropic waveguide is not aligned with its waveguide coordinate system in the plane defined by the propagation axis and the normal of waveguide surface.

## I. INTRODUCTION

LOSS reduction in power coupling between two waveguides has been one of the most important problems in light transmission optics. Butt-joining is a simple method widely used in connecting waveguides different in structure. Coupling efficiency with this method critically depends on both axial placement and angular alignment. Up to now a lot of practical junctions have been investigated theoretically and experimentally: fiber splices [1]–[5], coupling from light sources into fibers or channel-waveguides [6]–[8], coupling between fibers and/or channel-waveguides [9]–[11], etc.

Most optical components and integrated circuits have been fabricated on anisotropic crystalline materials. Higher coupling efficiency in butt-joints between fibers and channel waveguides will be increasingly required for future use of pigtailed optical devices in practical applications. To the authors' knowledge, however, there are few studies on butt-joints between fibers and channel waveguides consisting of anisotropic materials; no work has been reported which includes misalignment between the waveguide and material coordinate systems in the anisotropic waveguide. The angle between both coordinate axes is called oblique angle hereafter. Only Marcuse has predicted that an appropriate amount of angular misalignment between two waveguide axes would increase the power coupled under such a situation [12].

In the present paper, power-coupling efficiency is estimated for a butt-joint between an isotropic slab waveguide and an anisotropic one consisting of uniaxial crystalline material which has the optical axis in an arbitrary direction in the plane defined by the propagation axis and the normal of waveguide surface. Uncoupled TE and TM modes can be supported in slab waveguides with such oblique angles. We concentrate on analyzing the characteristics of TM modes because of their interesting behavior which cannot be observed in isotropic waveguides. Usually the power-coupling characteristics of more practical butt-joints between fibers and channel waveguides can be estimated almost directly from those for slab waveguides which are much simpler in structure and analysis.

## II. WAVEGUIDING STRUCTURE

An incoming isotropic slab waveguide (waveguide 1) with thickness  $d_1$  is butt-jointed to an outgoing anisotropic slab waveguide (waveguide 2) with thickness  $d_2$  as shown together with the coordinate systems used for the analysis in Fig. 1. The waveguide 1 is assumed to be symmetric. The waveguide 2 may be either symmetric or asymmetric. It is also assumed that only the dominant mode can be supported by each of these waveguides. Optical waves propagate along the  $z'$  axis in the waveguide 1 and along the  $z$  axis in the waveguide 2.  $\Delta$  is a displacement distance along the  $x$  axis and  $\theta$  a misalignment angle, in the  $x$ - $z$  plane, between the  $z$  and  $z'$  axes. In the following, subscripts 1 and 2 will be attached to the quantities belonging to the waveguides 1 and 2, respectively.  $\epsilon_{1p}$  and  $\hat{\epsilon}_{2p}$  whose additional subscript  $p$ , signifying the region, represents  $c$  (core) or  $d$  (cladding) are the scalar permittivity and the tensor permittivity, respectively.  $\epsilon_0$  is the free-space permittivity.

Suppose optical axes, in both core and cladding of the waveguide 2, are contained in the  $x$ - $z$  plane and are parallel to each other. In the waveguide coordinate system, the permittivity tensor is then expressed as

$$\hat{\epsilon}_{2p} = \begin{pmatrix} \epsilon_{xpp} & 0 & \epsilon_{xzp} \\ 0 & \epsilon_{ypy} & 0 \\ \epsilon_{xzp} & 0 & \epsilon_{zpp} \end{pmatrix} \quad (1)$$

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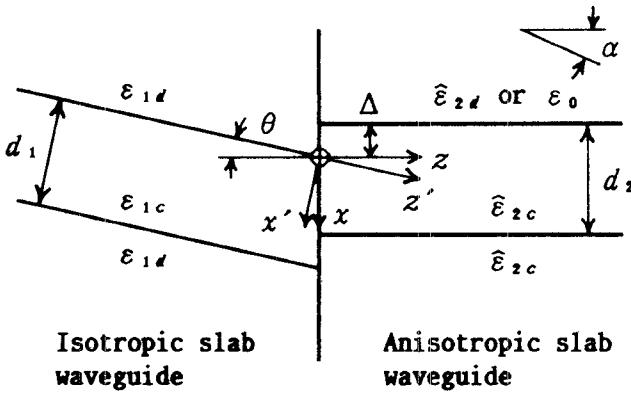


Fig. 1. Waveguide butt-joint and coordinate systems for the analysis.

where

$$\begin{aligned}\epsilon_{xip} &= \epsilon_{op} \cos^2 \alpha + \epsilon_{ep} \sin^2 \alpha \\ \epsilon_{xip} &= (\epsilon_{ep} - \epsilon_{op}) \sin \alpha \cos \alpha \\ \epsilon_{zip} &= \epsilon_{op} \sin^2 \alpha + \epsilon_{ep} \cos^2 \alpha \\ \epsilon_{yip} &= \epsilon_{op}.\end{aligned}\quad (2)$$

In the above equations,  $\epsilon_{op}$  and  $\epsilon_{ep}$  are the permittivities for the ordinary and extraordinary rays, respectively.  $\alpha$  is the oblique angle between the  $z$  axis and the optical axis. In general, angular misalignment makes a wedge-shaped air gap at the end-to-end junction. This gap, however, would be filled up with an index-matching liquid. Therefore, the butt-joint shown in Fig. 1 can be considered a good approximation to a practical butt-joint.

### III. ANALYTICAL METHOD

We concentrate our discussion on the TM mode propagation as mentioned before. In the case of the dominant mode incidence, we obtain

$$\begin{aligned}H_y^{(i)} + rH_y^{(r)} + \int_0^\infty r(\rho_1) H_y^{(r)}(\rho_1) d\rho_1 \\ = tH_y^{(t)} + \int_0^\infty t(\rho_2) H_y^{(t)}(\rho_2) d\rho_2\end{aligned}\quad (3)$$

$$\begin{aligned}E_x^{(i)} + rE_x^{(r)} + \int_0^\infty r(\rho_1) E_x^{(r)}(\rho_1) d\rho_1 \\ = tE_x^{(t)} + \int_0^\infty t(\rho_2) E_x^{(t)}(\rho_2) d\rho_2\end{aligned}\quad (4)$$

from the boundary conditions at the joint,  $z = 0$ . In the above equations, the time variation factor  $\exp(j\omega t)$  is suppressed. The superscripts  $(i)$ ,  $(r)$ , and  $(t)$  are added to

the incident, reflected, and transmitted waves, respectively.  $E_x$  is the  $x$  component of the electric field and  $H_y$  the  $y$  component of the magnetic field. The quantities represented as functions of  $\rho_1$  or  $\rho_2$ , the propagation constants in the transverse direction, belong to radiation modes and  $r$ ,  $t$ ,  $r(\rho_1)$ , and  $t(\rho_2)$  are unknown amplitude coefficients.

When  $\theta \neq 0$ , we can express the field components tangential to the  $z = 0$  plane in the waveguide 1 in terms of the field components described with the  $(x', y', z')$  coordinate system as follows:

$$H_y^{(i)} = H_{y'}^{(i)} \quad (5)$$

$$H_y^{(r)} = H_{y'}^{(r)} \quad (6)$$

$$E_x^{(i)} = E_{x'}^{(i)} \cos \theta + E_z^{(i)} \sin \theta \quad (7)$$

$$E_x^{(r)} = E_{x'}^{(r)} \cos \theta + E_z^{(r)} \sin \theta \quad (8)$$

where the mathematical description for the field components in the right-hand sides is summarized in Appendix I.

Usually, the refractive indices of two waveguides do not differ much for most fiber-to-channel butt-joints: about 1.5 in usual fibers consisting of fused silica and 2.27 or 2.2 in channel-waveguides made of  $\text{LiNbO}_3$ , the most common anisotropic bulk material used for optical devices. The interface of these two materials would reflect at most 4% of the incident power of plane wave for the normal incidence; less for incident TM waves tilted up to the Brewster angle. Judging from this fact, we can omit reflected radiation modes in (3) and (4) without causing any significant errors. After multiplying these equations by the complex conjugate of  $H_y^{(i)}$ , we integrate the resulting equations with respect to  $x$  over the infinite region. The orthogonality relations between the modes in the waveguide 2 finally lead to

$$\begin{aligned}\int \frac{1}{\epsilon_{xx}} H_y^{(i)} H_y^{(i)*} dx + r \int \frac{1}{\epsilon_{xx}} H_y^{(r)} H_y^{(r)*} dx \\ = t \int \frac{1}{\epsilon_{xx}} |H_y^{(t)}|^2 dx\end{aligned}\quad (9)$$

$$\begin{aligned}\int E_x^{(i)} H_y^{(i)*} dx + r \int E_x^{(r)} H_y^{(r)*} dx \\ = t \int E_x^{(t)} H_y^{(t)*} dx\end{aligned}\quad (10)$$

where the asterisk indicates the complex conjugate. Expressing  $E_x$  in  $H_y$  with the help of Maxwell's equations and applying the weakly-guiding approximation, we obtain

$$t = \frac{\frac{1}{\epsilon_{1d}} \left[ \int h_1 H_y^{(i)*} dx \int H_y^{(r)} H_y^{(i)*} dx - \int H_y^{(i)} H_y^{(i)*} dx \int h_2 H_y^{(i)*} dx \right]}{\left[ \frac{\epsilon_{xzd}}{\eta_d} \int H_y^{(r)} H_y^{(i)*} dx \int h_3 H_y^{(i)*} dx + \beta_2 \frac{\epsilon_{zdd}}{\eta_d} \int H_y^{(r)} H_y^{(i)*} dx \int |H_y^{(i)}|^2 dx - \frac{1}{\epsilon_{1d}} \int h_2 H_y^{(i)*} dx \int |H_y^{(i)}|^2 dx \right]} \quad (11)$$

where

$$\eta_d = \epsilon_{xxd} \epsilon_{zzd} - \epsilon_{xd}^2. \quad (12)$$

In the above equations,  $\beta_1$  and  $\beta_2$  are the propagation constants of the dominant mode and  $h_1$ ,  $h_2$ , and  $h_3$  are given in Appendix II.

We here define the power-coupling coefficient through the joint as the ratio of the power of the transmitted dominant mode to the power of the incident mode, that is,

$$T = |t|^2 \frac{\frac{\beta_2}{\epsilon_{xxd}} \int |H_y^{(r)}|^2 dx}{\frac{\beta_1}{\epsilon_{1d}} \int |H_y^{(i)}|^2 dx'}. \quad (13)$$

#### IV. RESULTS OF CALCULATIONS

We will present here some numerical results for two typical cases. The outgoing waveguide, which is assumed to be consisting of  $\text{LiNbO}_3$ , is symmetric in the first and is asymmetric in the second. The refractive indices of the incoming waveguide are chosen so that  $n_{1c} = (\epsilon_{1c}/\epsilon_0)^{1/2} = 1.51275$  and  $n_{1d} = (\epsilon_{1d}/\epsilon_0)^{1/2} = 1.51$ . These are representative values for optical fibers in practical use. The slab thickness is assumed as  $d_1 = d_2 = 5\lambda_0$  in both incoming and outgoing waveguides where  $\lambda_0$  is a free space wavelength. In the case that the outgoing waveguide has a graded index profile, a multi-layer approximation is applied to the modal analysis [13].

##### A. Symmetric Index Profile

Index profiles, whether graded or not, would be almost symmetric in waveguides buried in the deep inside of substrates by means of special fabrication processes. Consider two typical symmetric index profiles in the waveguide 2. The first is a step index profile expressed as

$$n_o(x) = \begin{cases} n_{od}, & x < 0 \\ n_{od} + \delta n_o, & 0 \leq x \leq d_2 \\ n_{od}, & d_2 < x \end{cases}$$

$$n_e(x) = \begin{cases} n_{ed}, & x < 0 \\ n_{ed} + \delta n_e, & 0 \leq x \leq d_2 \\ n_{ed}, & d_2 < x. \end{cases} \quad (14)$$

The second is a square-law index profile described as

$$n_o(x) = \begin{cases} n_{od}, & x \leq 0 \\ n_{od} + \delta n_o \left[ 1 - \left( \frac{x - d_2/2}{d_2/2} \right)^2 \right], & 0 \leq x \leq d_2 \\ n_{od}, & d_2 \leq x \end{cases}$$

$$n_e(x) = \begin{cases} n_{ed}, & x < 0 \\ n_{ed} + \delta n_e \left[ 1 - \left( \frac{x - d_2/2}{d_2/2} \right)^2 \right], & 0 \leq x \leq d_2 \\ n_{ed}, & d_2 < x. \end{cases} \quad (15)$$

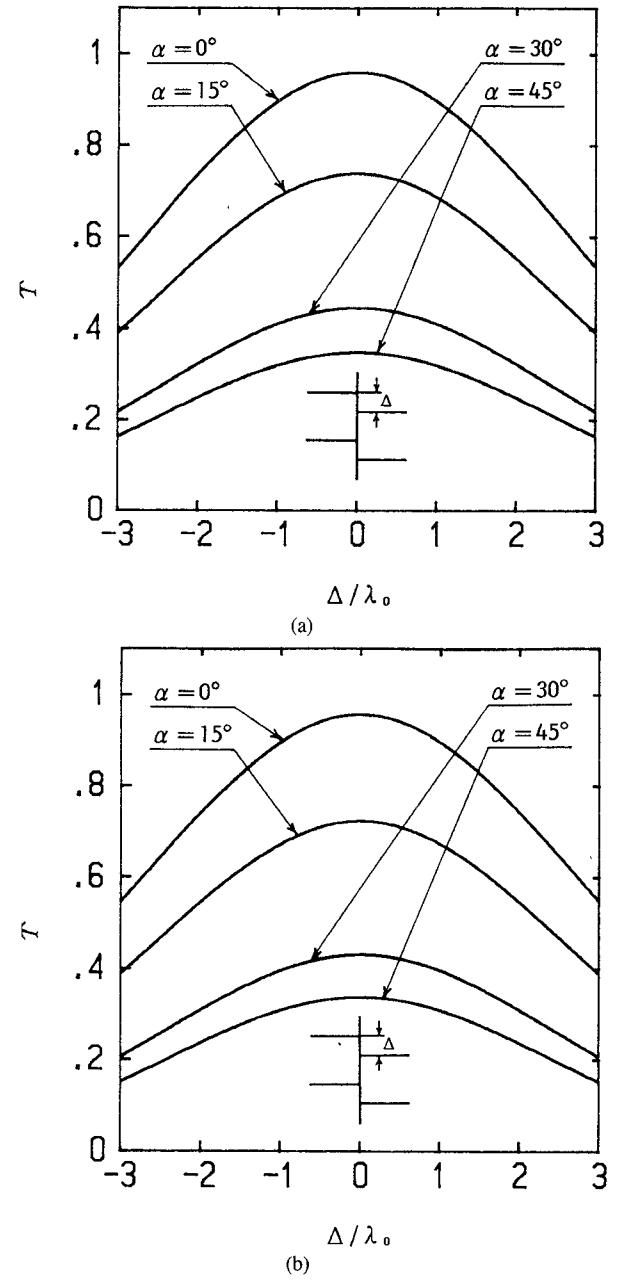


Fig. 2. Power-coupling coefficient versus normalized displacement distance with the oblique angle as a parameter. The outgoing waveguide has a symmetric structure: (a) step index profile and (b) square-law index profile. The structural parameters are chosen as  $d_1 = d_2 = 5\lambda_0$ ,  $n_{1c} = 1.51275$ ,  $n_{1d} = 1.51$ ,  $n_{od} = 2.272$ ,  $n_{ed} = 2.187$ ,  $\delta n_o = \delta n_e = 0.002$ .

The refractive indices are chosen as  $n_{od} = (\epsilon_{od}/\epsilon_0)^{1/2} = 2.272$ ,  $n_{ed} = (\epsilon_{ed}/\epsilon_0)^{1/2} = 2.187$ , and  $\delta n_o = \delta n_e = 0.002$  in the numerical calculations.

Fig. 2 shows curves for the power-coupling coefficients as a function of the normalized displacement distance  $\Delta/\lambda_0$  with the oblique angle  $\alpha$  as a parameter. It is assumed in these cases that there is no angular misalignment between the two waveguide axes. Power coupling strongly depends on the oblique angle as well as the displacement distance whether the index profile of the waveguide 2 is graded or not.

The power-coupling coefficients are shown in Fig. 3 as a function of misalignment angle  $\theta$  with the oblique angle

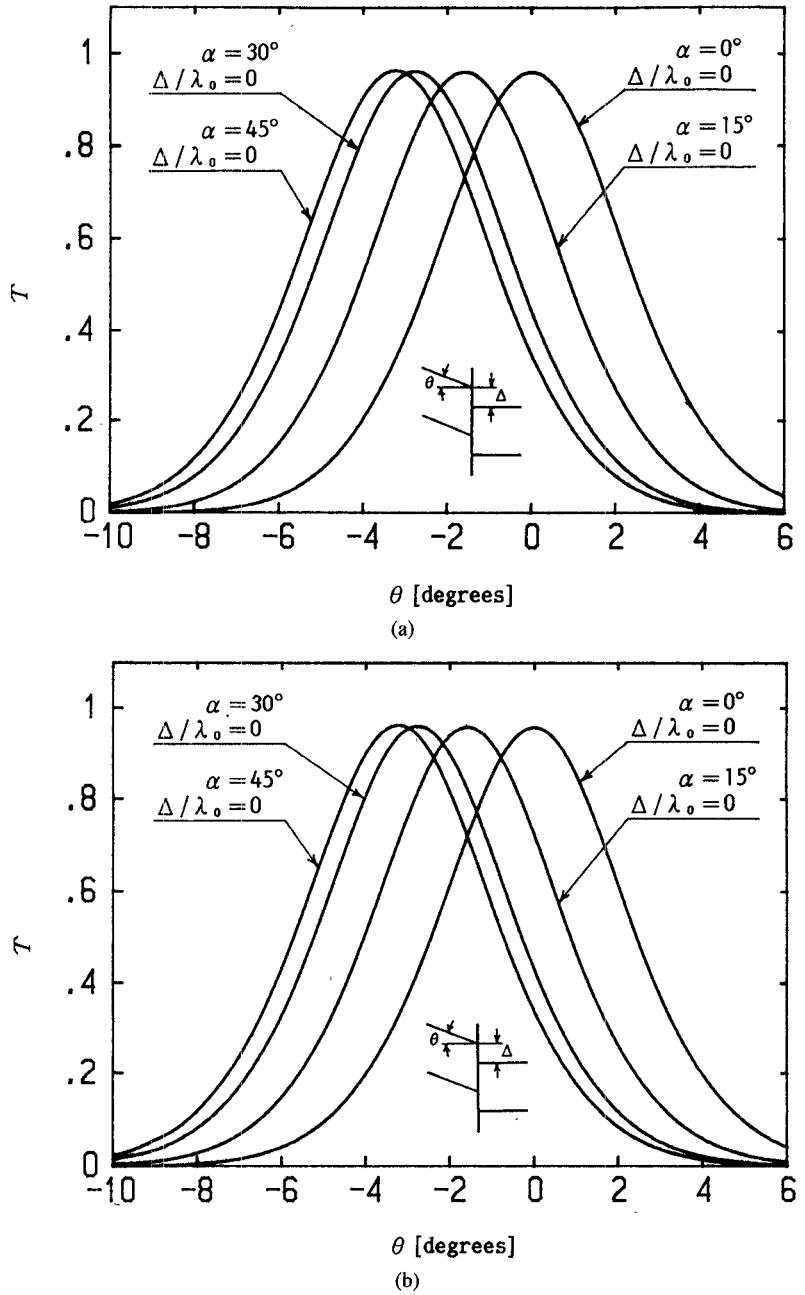


Fig. 3. Power-coupling coefficient versus misalignment angle with the oblique angle and the normalized displacement distance as parameters. The outgoing waveguide has a symmetric structure: (a) step index profile and (b) square-law index profile. The structural and material parameters are the same as in Fig. 2.

and the normalized displacement distance as parameters. The normalized  $\Delta$ 's in the figures are so optimally adjusted that the peak values become maximum. In these examples for the symmetric profiles, all of them are nearly equal to zero though. In the anisotropic waveguide, the misalignment in the waveguide and material coordinate systems causes the phase front to tilt in proportion to the oblique angle [12]. The values of  $\theta$  at which the curves reach their peaks are almost identical to the tilt angles of phase fronts determined by the oblique angles written down in the figures. Consequently it is understood that close agreement of phase fronts in both incoming and outgoing waveguides is a principal factor for achieving high efficiency in power coupling at a butt-joint.

#### B. Asymmetric Index Profile

In general, index profiles are strongly asymmetric in practical waveguides by means of such usual fabrication techniques as diffusion processes or ion-exchange processes. We consider two representative index profiles also in this case. The first is a step-index profile

$$n_o(x) = \begin{cases} 1, & x < 0 \\ n_{od} + \delta n_o, & 0 \leq x \leq d_2 \\ n_{od}, & d_2 < x \end{cases}$$

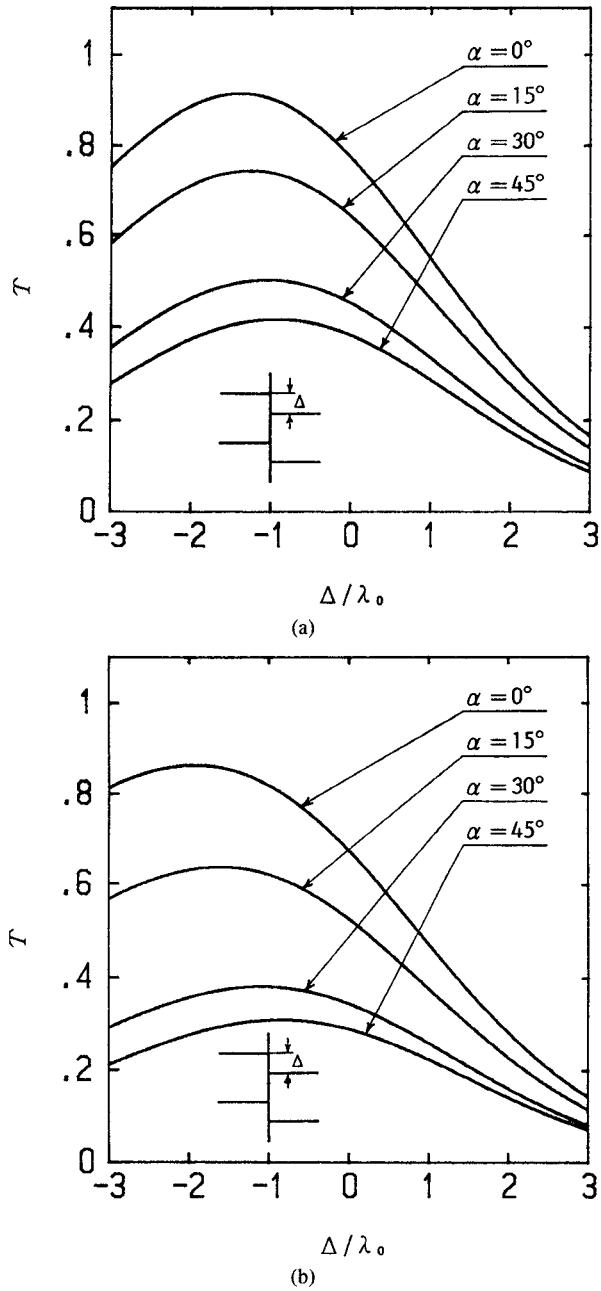


Fig. 4. Power-coupling coefficient versus normalized displacement distance with the oblique angle as a parameter. The outgoing waveguide has an asymmetric structure: (a) step index profile and (b) square-law index profile. The structural parameters are the same as in Fig. 2.

$$n_e(x) = \begin{cases} 1, & x < 0 \\ n_{ed} + \delta n_e, & 0 \leq x \leq d_2 \\ n_{ed}, & d_2 < x \end{cases} \quad (16)$$

$$n_e(x) = \begin{cases} 1, & x < 0 \\ n_{ed} + \delta n_e[1 - (x/d_2)^2], & 0 \leq x \leq d_2 \\ n_{ed}, & d_2 < x \end{cases}$$

and the second is a square-law index profile

$$n_o(x) = \begin{cases} 1, & x < 0 \\ n_{od} + \delta n_o[1 - (x/d_2)^2], & 0 \leq x \leq d_2 \\ n_{od}, & d_2 < x \end{cases}$$

The refractive indices are chosen to be the same as before.

Fig. 4 illustrates the effects of axial displacement on the power-coupling coefficients. The abscissa represents the normalized displacement distance. The oblique angle is taken as a parameter. Each curve reaches its peak at

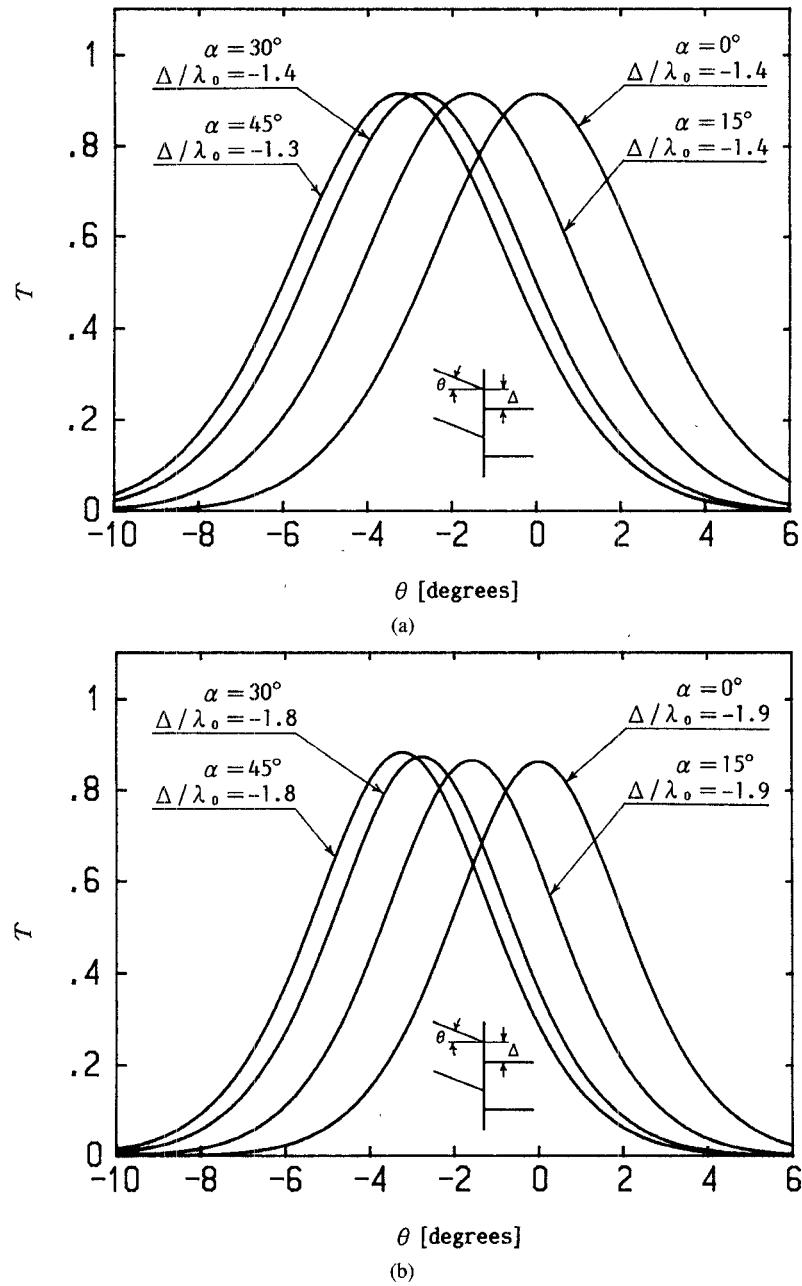


Fig. 5. Power-coupling coefficient versus misalignment angle with the oblique angle and the normalized displacement distance as parameters. The outgoing waveguide has an asymmetric structure: (a) step index profile and (b) square-law index profile. The structural and material parameters are the same as in Fig. 2.

other than  $\Delta/\lambda_0 = 0$  because of asymmetry in the field profile caused by the asymmetric waveguide structure.

Finally, Fig. 5 shows curves for the power-coupling coefficients as a function of the misalignment angle with the oblique angle and the normalized displacement distance as parameters. Each one of normalized  $\Delta$ 's in the figures is optimally adjusted also in the present case. They are other than zero and slightly different from each other. An appropriate misalignment angle dramatically raises the efficiency of power coupling. Asymmetric index profiles support modes of asymmetric field profile, which must

degrade the field overlapping between the two waveguide modes. This is the reason why each peak is a little bit lower than the corresponding peak of the former examples.

It is found from Figs. 4 and 5 that step index profiles show superior transmission characteristics to graded index profiles. A brief study on modal fields supported by asymmetric index profiles tells us that the field deformation from the symmetric case is more significant in graded index profiles; the spot-size is broader and the field distribution is less symmetric. Therefore, modal fields of step

index profiles match better with a symmetric field distribution of the incoming waveguide. This leads to the difference of the characteristics illustrated in the figures.

## V. CONCLUSION

Power-coupling efficiency is analyzed for a butt-joint between an isotropic slab waveguide and an anisotropic one where the material coordinate system is not aligned with the waveguide coordinate system in the plane defined by the normal of waveguide surface and the propagation axis. The formula for power-coupling coefficients is derived from the boundary conditions at the interface between the incoming and outgoing waveguides by neglecting reflected radiation modes. A number of numerical results are presented for various values of the oblique angle, displacement distance, and misalignment angle. It is found that the misalignment of the material and waveguide coordinate systems in the anisotropic waveguide exceedingly lowers the power transmitted through the joint. It is also found that appropriate amounts of angular misalignment and axial displacement between the two waveguides improve power-coupling efficiency remarkably whether the index profile of outgoing waveguide is symmetric or not, and whether graded or not.

## APPENDIX I

For the TM modes, the transverse magnetic field component of in the waveguide 1 is expressed as

$$H_{y'} = \begin{cases} A \exp(\gamma_1 x') \exp(-j\beta_1 z'), & x' < 0 \\ [A \cos(\kappa_1 x') + B \sin(\kappa_1 x')] \exp(-j\beta_1 z'), & 0 \leq x' \leq d_1 \\ [A \cos(\kappa_1 d_1) + B \sin(\kappa_1 d_1)] \\ \cdot \exp[-\gamma_1(x' - d_1)] \exp(-j\beta_1 z'), & d_1 < x' \end{cases} \quad (A1)$$

where

$$\begin{aligned} \kappa_1 &= [n_{1c}^2 k_0^2 - \beta_1^2]^{1/2} \\ \gamma_1 &= [\beta_1^2 - n_{1d}^2 k_0^2]^{1/2} \\ k_0 &= \omega(\epsilon_0 \mu_0)^{1/2} \end{aligned} \quad (A2)$$

$$B = \frac{\gamma_1 n_{1c}^2}{\kappa_1 n_{1d}^2} A. \quad (A3)$$

“A” is an arbitrary constant and the eigenvalue equation is

$$\tan(\kappa_1 d_1) = \frac{2\kappa_1 \gamma_1 n_{1c}^2 n_{1d}^2}{\kappa_1^2 n_{1d}^4 - \gamma_1^2 n_{1c}^4}. \quad (A4)$$

With the help of the coordinate transformation

$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x' \\ z' \end{pmatrix} \quad (A5)$$

we obtain from (A1)

$$H_{y'} = \begin{cases} A \exp[\gamma_1(x \cos \theta + z \sin \theta)] \\ \cdot \exp[-j\beta_1(-x \sin \theta + z \cos \theta)], & x < 0 \\ \{A \cos[\kappa_1(x \cos \theta + z \sin \theta)] \\ + B \sin[\kappa_1(x \cos \theta + z \sin \theta)]\} \\ \cdot \exp[-j\beta_1(-x \sin \theta + z \cos \theta)], & 0 \leq x \leq d_1/\cos \theta \\ [A \cos(\kappa_1 d_1) + B \sin(\kappa_1 d_1)] \\ \cdot \exp[-\gamma_1(x \cos \theta + z \sin \theta - d_1)] \\ \cdot \exp[-j\beta_1(-x \sin \theta + z \cos \theta)], & d_1/\cos \theta < x. \end{cases} \quad (A6)$$

Finally, the relationships between the electric and magnetic fields

$$\begin{aligned} E_{x'} &= \frac{j}{\omega \epsilon_{1p}} \frac{\partial H_{y'}}{\partial z'} \\ E_{z'} &= \frac{-j}{\omega \epsilon_{1p}} \frac{\partial H_{y'}}{\partial x'} \end{aligned} \quad (A7)$$

and the chain rule of differentiation lead to

$$\begin{aligned} E_{x'} &= \frac{j}{\omega \epsilon_{1p}} \left( -\sin \theta \frac{\partial}{\partial x} + \cos \theta \frac{\partial}{\partial z} \right) H_{y'} \\ E_{z'} &= \frac{-j}{\omega \epsilon_{1p}} \left( \cos \theta \frac{\partial}{\partial x} + \sin \theta \frac{\partial}{\partial z} \right) H_{y'}. \end{aligned} \quad (A8)$$

## APPENDIX II

$$h_1 = -2j \sin \theta \cos \theta \frac{\partial H_{y'}^{(i)}}{\partial x} + j(\cos^2 \theta - \sin^2 \theta) \frac{\partial H_{y'}^{(i)}}{\partial z} = \begin{cases} A[\beta_1 \cos \theta - j\gamma_1 \sin \theta] \exp(\gamma_1 x \cos \theta) \\ \cdot \exp(j\beta_1 x \sin \theta), & x < 0 \\ \{\beta_1 \cos \theta [A \cos(\kappa_1 x \cos \theta) + B \sin(\kappa_1 x \cos \theta)] \\ - j\kappa_1 \sin \theta [-A \sin(\kappa_1 x \cos \theta) \\ + B \cos(\kappa_1 x \cos \theta)]\} \exp(j\beta_1 x \sin \theta), & 0 \leq x \leq d_1/\cos \theta \\ [\beta_1 \cos \theta + j\gamma_1 \sin \theta] [A \cos(\kappa_1 d_1) \\ + B \sin(\kappa_1 d_1)] \exp[-\gamma_1(x \cos \theta - d_1)] \\ \cdot \exp(j\beta_1 x \sin \theta), & d_1/\cos \theta < x \end{cases} \quad (A9)$$

$$\begin{aligned}
 h_2 = & -2j \sin \theta \cos \theta \frac{\partial H_y^{(r)}}{\partial x} + j(\cos^2 \theta - \sin^2 \theta) \frac{\partial H_y^{(r)}}{\partial z} \\
 & - A[\beta_1 \cos \theta + j\gamma_1 \sin \theta] \exp(\gamma_1 x \cos \theta) \\
 & \cdot \exp(-j\beta_1 x \sin \theta), \quad x < 0 \\
 & - \{\beta_1 \cos \theta [A \cos(\kappa_1 x \cos \theta) \\
 & + B \sin(\kappa_1 x \cos \theta)] \\
 & + j\kappa_1 \sin \theta [-A \sin(\kappa_1 x \cos \theta) \\
 & + B \cos(\kappa_1 x \cos \theta)]\} \exp(-j\beta_1 x \sin \theta), \\
 & 0 \leq x \leq d_1/\cos \theta \\
 & [-\beta_1 \cos \theta + j\gamma_1 \sin \theta] [A \cos(\kappa_1 d_1) \\
 & + B \sin(\kappa_1 d_1)] \exp[-\gamma_1(x \cos \theta - d_1)] \\
 & \cdot \exp(-j\beta_1 x \sin \theta), \quad d_1/\cos \theta < x
 \end{aligned} \tag{A10}$$

$$h_3 = j \frac{\partial H_y^{(r)}}{\partial x}. \tag{A11}$$

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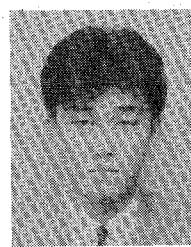
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